ner described above, then it is possible to complete the figure of the fifteen points *LM*, *LN*, ...*QR* and of the twenty lines *LMN*, *LMP*, ...*PQR*, such that through each point pass four lines, and on each line lie three points, as detailed in the foregoing.

Of the fifteen points, nine, viz. the points *LP*, *LQ*, *LR*; *MP*, *MQ*, *MR*; *NP*, *NQ*, *NR* are, as appeared above, points on two of the six lines 1, 2, 3, 4, 5, 6; the remaining points are *MN*, *NL*, *LM*; *QR*, *RP*, *PQ*. These are *Brianchon points*

MN	of the six-side	162435
NL	"	152634
LM	"	142536
QR	"	152436
RP	"	142635
PQ	"	162534,

for the point *MN* is the meet of lines *MNP*, *MNQ*, *MNR*≡*MP*, *NP*; *MQ*, *NQ*; *MR*, *NR*≡35, 26; 16, 34; 24, 15; that is, *MN* is the Brianchon point of the six-side 162435; and similar reasoning verifies the above statements for the rest of the six-lines.

To summarize, we have two sets of three six-sides such that the Brianchon points of each set lie *in linea;* and the two lines so obtained together with the eighteen lines through the six Brianchon points, form a system of twenty lines passing by fours through fifteen points.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

154. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Deduce the Sylvestrian Reciprocant of $ax^3+3bx^2y^2+ay^3+d=0$.

Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Differentiating, dividing by 3, and combining, we have

$$a(x^2+y^2\frac{dy}{dx})+2bxy(y+x\frac{dy}{dx})=0\ldots.(1).$$

Repeating the operation, we have

$$a\left[x+2y\left(\frac{dy}{dx}\right)^2+y^2\frac{d^2y}{dx^2}\right]+2b\left[y^2+4xy\frac{dy}{dx}+x^2\left(\frac{dy}{dx}\right)^2+x^2y\frac{d^2y}{dx^2}\right]=0\ldots(2).$$

Eliminating a and b in equations (1) and (2), we have

$$\begin{vmatrix} x^2+y^2\frac{dy}{dx}, & xy\left(y+x\frac{dy}{dx}\right) \\ x+2y\left(\frac{dy}{dx}\right)^2+y^2\frac{d^2y}{dx^2}, & \left(y+2x\frac{dy}{dx}\right)^2+x^2y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2 \end{vmatrix}=0,$$

which is the Sylvestrian Reciprocant of $ax^3+3bx^2y^2+ay^3+d=0$, since this function would have the same form if x were the dependent and y the independent variable.

167. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A weight of m pounds falls and is broken into n pieces after which it is found that all weights, in pounds, from 1 to m can be weighed. Find the weight of each piece. Apply when $m=121$, $n=5$.

I. Solution by the PROPOSER.

Let x_1, x_2, x_3, ...x_n be the n pieces. Then $x_1+x_2+x_3+\ldots.+x_{n-1}+x_m=m$, $x_2-x_1=x_1+1$, or $x_2=2x_1+1$.

$x_3-x_2-x_1=x_2+x_1+1$, or $x_3=2x_2+2x_1+1=3(2x_1+1)=3x_2$.

$x_4-x_3-x_2-x_1=x_3+x_2+x_1+1$, or $x_4=2x_3+3x_2=9(2x_1+1)=9x_2$.

Generally, $x_r=3^{r-2}(2x_1+1)=3^{r-2}x_2$.

$\therefore x_1+x_2+x_3+\ldots.+x_n=x_1+(2x_1+1)(1+3+9+27+\ldots.+3^{n-2})=m$.

$\therefore x_1+(2x_1+1)(3^{n-1}-1)=2m$ or $x_1\dfrac{2m+1-3^{n-1}}{2.3^{n-1}}$.

$x_2=(2x_1+1)=\dfrac{2m+1}{3^{n-1}}$; $x_r=3^{r-2}x_2=\dfrac{2m+1}{3^{n-r+1}}$.

When $m=121$, and $n=5$, x_1-1, $x_2=3$, $x_3=9$, $x_4=27$, $x_5=81$.

II. Solution by FRANK L. GRIFFIN, Graduate Student, The University of Chicago.

Let $f(n)$=number of groupings of n weights in two groups; then the maximum number giving one group a preponderence is $f(n)/2$.

Now, $f(n)=3^n-1\ldots.(i)$ [for proof see below]. Hence, to weigh all weights, in pounds, from 1 to m by using n weights, it is necessary that

$$m\overline{\overline{<}}\frac{3^n-1}{2}\ldots.(ii).$$

By using the n weights, 1, 3, 9, ...3^{n-1}, all weights, in pounds, from 1 to